Enhanced Channel Estimation comparison Based On Basis Expansion Using Slepian Sequences for Time Varying OFDM Systems

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Abstract

Successful communication always depend upon the maximum data extraction from the received data. As the data travels through the channel it undergoes Doppler shift. Thus channel estimation has become major issue in data transmission since the mobile equipment have become fast moving devices. Intersymbol interferences (ISI), Inter Carrier Interferences (ICI) are the major data disturbances.

In this paper we compare the channel estimation using traditional cyclic prefix and unique word method. Here we have used Basis expansion modeling (BEM) for channel estimation. This method can model varying channel in mobile scenario. The least square and MMSE technique both are compared for their number of calculations and perfection in estimation. The Discrete prolate spheroidal sequence improves the channel estimation by sending frequency modulated cosine as pilot. The slepian sequences have maximum energy concentration in each subcarrier band. The Prolate function shows that the BER can be effectively reduced by finding maximum delay spread and maximum Doppler spread. It does not depend upon particular channel scattering function. We can also reduce the dependency on finding all the channel delay paths.

Keywords: CP (Cyclic Prefix), UW (Unique Word), BEM(Basis Expansion Mode), LS(Least square), MMSE(Minimum Mean square error)

I. INTRODUCTION

With increase in requirement of data rate OFDM became more popular with its higher data rates. Splitting the data over multiple carriers also reduces the effective symbol rate over one sub-carrier. All the sub-carriers are orthogonal to each other. The convolution if comes zero, the signals are called orthogonal to each other. It means that there are no similar components in two consecutive carriers. This orthogonality provides the facility of overlapping the sub-carriers partially. Thus this overlapped sub-carriers are used twice. Thus spectral efficiency increases. OFDM provides good ICI protection. With inversion of guard band we can achieve good ISI also.

As the data travels over the channel it undergoes path delay. This increases symbol duration. Thus within the symbol duration channel may vary with time. This affect the amplitude of data. Thus it becomes necessary to estimate the channel over time. As the data travels, it gets spread and travels through scattering over various path. This data experiences Doppler frequency shift. In fast time varying channel thus, variation occurs within single OFDM symbol with time and frequency both. This is called doubly selective channel. As the Doppler frequency shift occurs orthogonality between adjacent channel will be no more. With numerous path the symbol duration increases and overlaps upon adjacent channel. Thus ICI and ISI both occur. This makes data extraction difficult at receiver side. This is because non zero value at other subcarriers. To avoid this guard band (cyclic prefix) are added. These CP are nothing but repetition of the end of symbol at the start of each symbol. This avoids the effect of multi path delay. In this case channel estimation and compensation becomes important. There are various methods of channel estimation. It is now become essential to do the comparative study of these techniques with their impact on Bit error rate given signal to Noise ratio.

In OFDM the binary data is first modulated using 16 QAM and then sampled at niquist rate. The two subcarriers are orthogonal thus all sampled subcarriers are summed to form OFDM sample. Now it is oscillator that can generate various frequency sub-carriers with coherent demodulator. This can be achieved by FFT operation.

To start with estimation of channel we require the information about amplitude and path delay spread. The two types of arrangements are done in channel estimation. In block type arrangement pilots are time multiplexed where in comb type pilots are frequency multiplexed. In highly faded environment where channel varies faster than symbol channel estimation at the receiver becomes challenging for two main reasons 1) the receiver needs to perform this estimation more frequently and 2) channel time-variations introduce inter-carrier interference among the OFDM sub-carriers which can degrade the performance of conventional channel estimation algorithms.

II. PROBLEM DEFINITION

A. Channel modelling

To extract the maximum data transmitted through the channel that varies with time and frequency it is necessary to generate channel matrix. Basis Expansion Models (BEM) can map the channel block by block basis. Also the method does not depend upon the previous estimation[12]. With each time-varying convolution filter coefficient modelled as a linear combination of certain basis functions.

B. Channel estimation

Convolutional operator can estimate only those channels which do not vary over symbol time duration and only gives multipath delay spread. This estimation is limited upto finding the Doppler spread. At receiver side the data is sampled with time duration Δt . For the increased sampling rate estimation becomes accurate at the cost of increased over the system.

In doubly selective channels, basis expansion model is useful. In BEM discrete channel taps are modelled as time-varying functions, thus the BEM models a doubly selective channel as a time varying filter. With the BEM, the channel taps are approximated by linear combinations of prescribed basis functions.

C. Goals

The estimation of channel require the information about multi path delay spread, amplitude information.[12] The channel estimation technique is required to fulfil the following goals A numerically stable algorithm with minimum calculation. The estimation should be able to work in frequency domain and time domain both. Better results with comparatively less complexity Technique should be independent of number of delay spread and their associated Doppler frequency shift.

D. Computation of the Discrete Prolate Spheroidal Window

The received signal is sampled at the receiver to extract the data. But sampled at ΔT duration can not extract complete data. Thus multi tapper method is used here data is sampled with multiple taps using slepian sequences. This sequence is highly band limited within the band {-w to +w}. The slepian sequences are found by using eigenvector. Only those eigen values(λ) with maximum energy are considered in eigen vector. The Eigen value ranges between 0 to 1. Thus maximum energy λ is at zeroth order of Discrete prolate spheroidal window (DPSW). The DPSW is a scalar multiple of the zero-order DPSS for a given N (length) and W(band), and is given by an eigenvector associated with the dominant eigenvector of the matrix T(N,W).

Example of DPSW



Figure 1 DPSW for N = 31 and W = 0.1 a) Time Domain b) Frequency domain

III.OFDM Model

A. OFDM Model

In OFDM baseband transmit signal sets the transmit pulse g to a rectangular window with support equal to the symbol duration T, i.e, g=x[0,T]

Thus the baseband signal is given by

$$s(t) = \sum_{t=-\infty}^{\infty} \sum_{k=0}^{k-1} al, k X[0,T] e^{j2\pi k f a(t-lT)}$$
(1)

If the channel response varies faster than one symbol time period then symbol is given as $x(t) = \sum_{k=0}^{k-1} A(k) e^{j2\pi k f s t}$

(2)

$t \in [0,T]$

Each sub-carrier will now transmit one symbol A(k) of length *l*. To convert this data in time domain so that pilots can be inserted IDFT is taken. $x(t) = \frac{1}{\sqrt{k}} \sum_{k=0}^{k-1} A(k) e^{j2\pi nk/K}$

(3)

Now x is time domain with n discrete time

IV. Simulation Results

Various estimation techniques of channel using pilot are compared in this paper.

A. Cyclic Prefix

In cyclic prefix we repeat the tail of symbol at the start. This frequency domain vector is then transformed into time domain via the IDFT operation, which we denote by a matrix operation x = F-1 N \tilde{x} utilizing the N-point DFT matrix FN with the element of the m-th row and the n-th column

$$[FN]m,n = e^{-J2\pi mn/N}$$
, (4)

where m, n = 0, 1, 2, ..., N - 1.

The guard interval is then formed by copying the last values to the front.

Figure 2 shows the bit error rate for various signal to noise ratios. This result is obtained for the following set of values.

OFDM Symbol	m=128
Length of symbols	N=64
Constallation map	M=16
Pilot frequence	4
Pilot energy	E=1
CP length	NCP=64
Channel length	L=2

Table 1 Set of OFDM channel parameters

The Cyclic prefix is used as pilot in the guard band. This is pilot based channel estimation technique. The estimation is mapped on graph with SNR(dB) on horizontal axis and BER on vertical axis. Channel estimation using CP





From the graph 2 we can get the information that upto 20dB the BER is almost same . As SNR is increased above 20dB experiences very low BER.

B. Unique word

UW is part of the IDFT output, and therefore also part of the DFT interval TDFT

T _{GI} ₩	$T_{\rm DFT}$		$\xrightarrow{T_{\text{GI}}}$		FT		$\stackrel{T_{\mathrm{GI}}}{\longleftrightarrow}$	<u>(</u>
CP1	Data 1	CP1	CP2	Data	2	CP2	CP3	
T _{GI} ₩	$T_{\rm DFT}$		<i>(</i>	$T_{\rm DFT}$		<	$T_{\rm D}$	FT
		_						



UW is known to receiver thus additional processing can be done. This improves the estimation process. The loss of bandwidth is also avoided. The estimation is mapped on graph with SNR (dB)on horizontal axis and BER on vertical axis.

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Fig 4 Channel estimation using UW

From the graph 4 we can get the information that up to 20dB the BER reduces consistently. As SNR is increased above 20dB the channel experiences low BER.

No of Carriers	64		
Coding	Convolutional		
Frame size	96		
No. of frames	100		
QAM	16		
No of sequences in DPS	9600		
Padding	Zero		
Equalization	Zero frequency		
Decoding	Veterbi decoding		
Time half bandwidth	4		

Table 2. Parameters used for channel estimation with DPSS

Comparison between CP and UW

Unique word is calculated at transmitter. And also known to receiver, thus is better. The estimation mapped on graph with SNR(dB)on horizontal axis and BER on vertical axis. Figure 5 gives comparison of both techniques in graph.



Figure 5 no prolate CP vs UW

C.Basis Expansion Model

Due to multipath delay spread and Doppler frequency shift the channel matrix does not remain diagonal. Thus BEM using number of channel taps. For the general case of doubly-selective channels, which occur due to the multipath delay and the Doppler effect, the wireless channel act like a time-varying filter. In BEM time varying filter coefficients are expressed as combination of basis function. Thus the l^{th} discrete channel tap *hl* is modeled as:

$$hl = \sum_{l=0}^{L-1} blmBm$$

 $l = 0, \ldots, L - 1$

where {Bm} is the set of M basis functions used to express the channel taps, and L is the maximum discrete path delay. one OFDM symbol duration is studied. With L discrete channel taps, the algorithm has a computational complexity of $O(L^2)$ in operations and memory.

Channel estimation with Prolate spheroidal sequences. The DPSS's can be viewed (and derived) as the discrete-time, finite-length sequences whose Discrete-Time Fourier Transform (DTFT) is most concentrated within a given bandwidth. Most significantly. If sequence of length N and bandlimit W, the first \approx 2NW DPSS capture all of the energy. After that CP is added to IDFT signal. the DPSS representation. For Slepian sequences with CP estimation the parameters are selected as in table. And the graph of SNR Vs BER is shown in figure 6.



Fig 6 Channel estimation using CP with Prolate sequences

. Using CP as pilot along with Prolate function provides constant reduction in bit error which is almost negligible at high value of SNR of about 40dB. At the same time number of calculations are reduced from $O(N^3)$ to $O(N^2)$. The table 3 gives the parameters used for estimation using DPSS

Slepian seq.	BER
0.4189	0.4639
0.3271	0.4248
0.2100	0.3742
0.1765	0.3090
0.0761	0.2398
0.0231	0.1208
0.0004	0.0224
T 11 2 01	1 T T T T

Table 3.Slepian sequences with UW estimation

The value of parameters set for matlab simulation are given in table 3.Using UW as pilot along with Prolate function reduces the BER as the SNR is improved. The graph in figure 7 shows the SNR Vs BER using UW with DPSS.



Fig 7 Channel estimation using UW imposed on

(5)

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The graph shows improved BER at less SNR. Thus with reduced calculation, and reduced complexity we get better error rate if used slepian sequences with unique word.

LS estimation Consider n pts(X_i, Y_i) where X_i are independent and Y_i are dependent $f(X,\beta)$ is a function with β having m parameters. Out of m parameters best suited parameters are found to get minimum sum of squares.

(6)

(7)

$$r_i^2 = Yi - f(Xi, \beta)$$

MSE

Mean squared error for the weight estimate can be expressed as

$$MSE = tr\{E\{(\hat{\rho} - \rho)(\hat{\rho} - \rho)^H\}\}$$

Here the two methods 1. LS and 2. MMSE are compared. The estimation mapped on graph with SNR(dB)on horizontal axis and BER on vertical axis.



Figure 8 comparison between LS and MMSE.

It is clear from the graph 6.8 that MMSE estimation is more accurate as compared to LS estimation. But MMSE is more complicated than that of LS estimation.

V. Conclusion and Future scope

A. Conclusion

This Paper is estimation comparison of various techniques and their combinations with each other. Normally it is always known that as SNR increases BER decreases. But increase in SNR also increases power requirement. If we add pilot symbols which are recalled at the start significant symbol rate is decreases. It is clear from the results that pilot aided estimation with CP gives the BER which reduces as Signal to Noise ratio is increased. When replaced cyclic prefix with unique word it gives better BER. With the addition of unique word the better estimation is performed and the BER reduces as SNR increases rapidly as compared to CP. As unique word are also known to receiver estimation becomes accurate.

When combined channel estimation at receiver with Prolate spheroidal sequences, the BER was slightly increased. But the technique reduces complexity from $O(N^3)$ of State of art approach to $O(N^2)$. Also it was proved that the technique does not depend upon the path delay function. It was also observed that the 2D prolate function is independent of channel scattering function. DPSS are more concentrated at the center than the traditional rectangular pulses used as pilot. On each slepian sequence the BER reduces and at the seventh sequence BER is minimum.

When the channel is doubly selective (time selective and Frequency selective both) faded the Basis expansion model with space time block coding (STBC) is useful. The LS technique is simple but not as accurate as MMSE. But MMSE increases the complexity severely. The impulse response is sampled at receiver. But only one sampled sequence can't provide proper results. Thus multiple taps are used for estimation.

Future scope

The study of Pilot assisted channel estimation in Dissertation shows that increase in the number of pilots also improves estimation. There can be an modification of virtual channel that can work for estimation. Thus effectively improve the symbol rate also. It has to be considered that the channel can be water in underwater data transmission or sand in underground data transmission. Thus the analysis of channel using DPSS can be further continued with various different channels.

The Walsh-Hadamard transform (WHT) is an orthogonal transformation that decomposes a signal into a set of orthogonal, rectangular waveforms called Walsh functions. The transformation has no multipliers and is real because the amplitude of Walsh (or Hadamard) functions has only two values, +1 or -1. Therefore WHT can be used in many different applications, such as power spectrum analysis, filtering, processing speech and medical signals, multiplexing and coding in communications, characterizing non-linear signals, solving non-linear differential equations, and logical design and analysis.

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